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**Cambridge Assessment  
International Education**

# Cambridge International AS & A Level

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## FURTHER MATHEMATICS

**9231/42**

Paper 4 Further Probability & Statistics

**October/November 2024**

**1 hour 30 minutes**

You must answer on the question paper.

You will need: List of formulae (MF19)

### INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

### INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

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1 A scientist is investigating the lengths of the leaves of a certain type of plant. The scientist assumes that the lengths of the leaves of this type of plant are normally distributed. He measures the lengths,  $x$  cm, of the leaves of a random sample of 8 plants of this type. His results are as follows.

3.5 4.2 3.8 5.2 2.9 3.7 4.1 3.2

Find a 90% confidence interval for the population mean length of leaves of this type of plant. [4]





2 The random variable  $X$  has probability generating function  $G_X(t)$  given by

$$G_X(t) = \frac{1}{5} + pt + qt^2,$$

where  $p$  and  $q$  are constants.

(a) Given that  $E(X) = 1.1$ , find the numerical value of  $\text{Var}(X)$ .

[4]





The random variable  $Y$  has probability generating function  $G_Y(t)$  given by

$$G_Y(t) = \frac{2}{3}t \left(1 + \frac{1}{2}t^2\right).$$

The random variable  $Z$  is the sum of independent observations of  $X$  and  $Y$ .

(b) Find the probability generating function of  $Z$ . [2]

(c) Find  $P(Z > 2)$ . [1]

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(d) State the most probable value of  $Z$ . [1]

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3 Rosie sows 5 seeds in each of 150 plant pots. The number of seeds that germinate is recorded for each pot. The results are summarised in the following table.

| Number of seeds that germinate | 0  | 1  | 2  | 3  | 4  | 5 |
|--------------------------------|----|----|----|----|----|---|
| Number of pots                 | 12 | 40 | 43 | 35 | 16 | 4 |

Rosie suggests that the number of seeds that germinate follows the binomial distribution  $B(5, p)$ .

(a) Use Rosie's results to show that  $p = 0.42$ . [1]

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(b) Carry out a goodness of fit test, at the 10% significance level, to test whether the distribution  $B(5, 0.42)$  is a good fit for the data. [9]

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4 The random variable  $X$  has probability density function  $f$  given by

$$f(x) = \begin{cases} \frac{1}{21}(x-1)^2 & 2 \leq x \leq 5, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find the cumulative distribution function of  $X$ .

[3]

The random variable  $Y$  is defined by  $Y = (X-1)^4$ .

(b) Find the probability density function of  $Y$ .

[3]





(c) Find the median value of  $Y$ .

(d) Find  $E(Y)$ .

[2]





5 Dev owns a small company which produces bottles of juice. He uses two machines,  $X$  and  $Y$ , to fill empty bottles with juice. Dev is investigating the volumes of juice in the bottles. He chooses a random sample of 35 bottles filled by machine  $X$  and a random sample of 60 bottles filled by machine  $Y$ . The volumes of juice,  $x$  and  $y$  respectively, measured in suitable units, are summarised by

$$\sum x = 30.8, \quad \sum x^2 = 29.0, \quad \sum y = 62.4, \quad \sum y^2 = 76.8.$$

Dev claims that the mean volume of juice in bottles filled by machine  $Y$  is greater than the mean volume of juice in bottles filled by machine  $X$ . A test at the  $\alpha\%$  significance level suggests that there is sufficient evidence to support Dev's claim.

Find the set of possible values of  $\alpha$ .

[9]



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**Turn over**



6 A sports college keeps records of the times taken by students to run one lap of a running track. The population median time taken is 51.0 seconds. After a month of intensive training, a random sample of 22 new students run one lap of the track, giving times, in seconds, as follows.

|      |      |      |      |      |      |      |      |
|------|------|------|------|------|------|------|------|
| 51.3 | 52.0 | 53.4 | 49.2 | 49.3 | 51.1 | 52.2 | 47.2 |
| 53.0 | 48.5 | 49.4 | 50.3 | 50.8 | 51.6 | 49.1 | 52.3 |
| 51.8 | 52.4 | 47.9 | 48.9 | 50.6 | 51.9 |      |      |

It is claimed that the intensive training has led to a decrease in the median time taken to run one lap of the track.

Carry out a Wilcoxon signed-rank test, at the 5% significance level, to test whether there is sufficient evidence to support the claim. [9]

[9]



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## Additional page

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